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THE STEADY, RECTILINEAR TOWING OF A WEIGHTLESS HYDROFOIL-CABLE SYSTEM IN A NON-UNIFORM STREAM

EDO CORPORATION
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THE STEADY, RECTILINEAR TOWING OF A WEIGHTLESS HYDROFOIL-CABLE SYSTEM IN A NON-UNIFORM STREAM

by

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ABSTRACT

This report presents the solution of a problem involving the towing of a hydrofoil-cable system in a non-uniform stream. The system consists of a weightless hydrofoil connected to the towing vehicle by a weightless flexible cable. The towing vehicle moves in a straight line at constant speed through a fluid medium wherein the steady fluid velocity is parallel to the direction of motion but varies in magnitude in the normal direction. The problem consists in determining the system configuration and the cable tension for specified velocity conditions, cable dimensions and hydrofoil characteristics. It is shown that the solution of this two-dimensional problem involves the simultaneous solution of two transcendental equations, thus requiring the use of numerical and/or graphical methods. Finally, a numerical example is presented to illustrate the method.

NOTATION

A. PHYSICAL QUANTITIES

A	Representative (planform) area of hydrofoil
a	Ratio of stream velocity gradient to velocity of towing vehicle
b	Fluid velocity gradient (constant in numerical example)
c	Constant $(= k/T')$
C_D	Hydrofoil drag coefficient
C_L	Hydrofoil lift coefficient
C_R	Cable resistance coefficient (for cable normal to stream)
D	Hydrofoil drag force
D_c	Cable diameter
k	Reference normal cable loading $\left(= C_R \cdot \frac{\rho}{2} V_o^2 \cdot D_c\right)$
L	Hydrofoil lift force
N	Cable normal force
S	Total cable length
s	Running cable length
T	Cable tension
T'	Reference tension value $\left(= \sqrt{C_L^2 + C_D^2} \cdot A \cdot \frac{\rho}{2} V_o^2\right)$
V_o	Velocity of towing vehicle, relative to still water
V_s	Velocity of stream, relative to still water
x, y	Rectangular coordinates of cable element
ρ	Mass density of fluid
ϕ	Local inclination of cable curve

B. MATHEMATICAL QUANTITIES

F	See Eq. (15)
f	See Eq. (7)
G_1, G_{-1}	See Eq. (25)
Γ_1, Γ_{-1}	See Eq. (27)
g	See Eq. (22)
ψ_{-1}	$e^{-cF(y_1)/f(y_1)}$

C. SUBSCRIPTS

o	Junction of cable and towing vehicle (origin of coordinates)
1	Junction of cable and hydrofoil
a	Denotes value corresponding to constant value of ϕ_1
b	Denotes value corresponding to constant value of S

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SECTION 1

INTRODUCTION

1.1 BACKGROUND

Under ONR Contract NOnr-3582(00), a general analysis is to be made of the steady turning of cable-towed body systems, wherein the towing vehicle moves in a circular path at constant speed.

This is a relatively complex problem, involving several features never previously analyzed, as follows:

(1) A non-uniform field of flow;

(2) Motions and forces in three dimensions;

(3) The complicated relation between the motion of the towed body (as part of the system), its attitude, and the forces and moments acting upon it.

Accordingly, it was considered desirable to analyze a number of non-trivial problems involving one or more of these features in order to establish mathematical procedures for, and gain insight into, the physical aspects of the general turning problem.

This report covers an analysis of the first such problem established, involving a non-uniform flow field. In addition to its pertinence to the turning problem, the particular case analyzed herein also has appreciable intrinsic interest.

1.2 STATEMENT OF PROBLEM

A neutrally buoyant, completely submerged, smooth, round cable of specified diameter and length is used to tow a stable hydrofoil, as illustrated in figure 1. The hydrofoil and its attachment to the tow cable are arranged so that the hydrofoil operates at a fixed angle of attack and, consequently, at fixed (specified) lift and drag coefficients. The cable is towed by a vehicle which moves horizontally in a straight line (x-axis, negative x-direction) at a specified constant speed through a moving fluid. The fluid velocity is everywhere horizontal and its magnitude is a specified function of the y-coordinate. The problem then is to establish a procedure for determining the hydrofoil location relative to the towing vehicle, and the cable shape and (constant) tension, if the cable is subject only to a normal force per unit length proportional to the square of the local normal component of relative velocity between cable and fluid.

SECTION 2
METHOD OF SOLUTION

2.1 ANALYSIS

For system equilibrium, the entire cable and the hydrofoil must move with constant velocity, V_o, (relative to an inertial frame, or "still water") in the same direction as the towing vehicle. As shown in figure 1, this direction is taken as the negative x-direction. The (specified) stream velocity (relative to still water), taken as positive in the x-direction, can be written as $V_s(y)$. Hence, the fluid velocity relative to the hydrofoil-cable system is:

$$V\ (y) = V_o + V_s\ (y) \tag{1}$$

It is assumed first that the only hydrodynamic forces acting on the cable are normal forces, i. e. the frictional (tangential) forces are neglected. Further, the single equation defining the equilibrium of the cable element is given by Eq. (2) of Reference 1. In the present notation, this is:

$$T\ d\phi = dN \tag{2}$$

where: T = Cable tension (unknown constant)

ϕ = Angle of inclination of cable element (to x-axis)

dN = Normal force on cable element

In accordance with Reference 1, the hydrodynamic force, dN, is taken as proportional to the square of the normal component of the local relative fluid velocity, specifically:

$$dN = C_R \cdot \frac{\rho}{2}\ (V\ \sin\phi)^2\ D_c\ ds \tag{3}$$

Combining Eq. (2) and (3):

$$\frac{dN}{d\phi} = C_R \cdot D_c \cdot \frac{\rho}{2} V^2 \sin^2 \phi \frac{ds}{d\phi} = T \tag{4}$$

which can be written as:

$$k \cdot f(y) \cdot \sin^2 \phi \cdot \frac{ds}{d\phi} = T \tag{5}$$

where:

$$k = C_R \cdot \frac{\rho}{2} V_o^{\,2} \cdot D_c \tag{6}$$

and

$$f(y) = \left(\frac{V}{V_o}\right)^2 = \left[1 + \frac{V_s(y)}{V_o}\right]^2 \tag{7}$$

i. e., f(y) is a known function.

It is seen that Eq. (5) involves two (2) unknown functions, $y(\phi)$ and $s(\phi)$, as well as the unknown constant, T.

Now, the tension must equal the resultant hydrofoil load:

$$\left.\begin{aligned} T &= \sqrt{L^2 + D^2} \\ &= A\sqrt{C_L^{\,2} + C_D^{\,2}} \left(\frac{\rho}{2} V_1^{\,2}\right) \end{aligned}\right\} \tag{8}$$

wherein A, C_L and C_D are known quantities, but $V_1 = V(y_1)$ is unknown.

Eq. (8) can be written as:

$$T = T' \cdot f(y_1) \tag{9}$$

where T' is the known constant:

$$T' = A\sqrt{C_L^{\,2} + C_D^{\,2}}\left(\frac{\rho}{2}\, V_o^{\,2}\right) \tag{10}$$

and y_1 is the unknown total cable height.

Combining Eq. (5) and (9), and using the relation:

$$dy = (\sin\phi)\, ds, \tag{11}$$

there is obtained:

$$\frac{d\phi}{\sin\phi} = c \cdot \frac{f(y)}{f(y_1)}\, dy \tag{12}$$

where c is the known constant:

$$c = \frac{k}{T_1} = \frac{C_R}{\sqrt{C_L^{\,2} + C_D^{\,2}}} \cdot \frac{D_c}{A} \tag{13}$$

Taking the origin of coordinates (and of cable length) at the towing vehicle, as shown in figure 1, integration of Eq. (12) yields:

$$\ln\left(\frac{\tan\frac{\phi}{2}}{\tan\frac{\phi_o}{2}}\right) = c\,\frac{F(y)}{f(y_1)} \tag{14}$$

where ϕ_o is the cable inclination at the towing vehicle, and $F(y)$ is the integral:

$$F(y) = \int_o^y f(y)\, dy \tag{15}$$

As $f(y)$ is a known function, so is $F(y)$.

At the hydrofoil-cable junction:

$$\phi = \phi_1, \quad y = y_1 \tag{16}$$

Inserting these values in Eq. (14):

$$\ln\left(\frac{\tan\frac{\phi_1}{2}}{\tan\frac{\phi_o}{2}}\right) = c\,\frac{F(y_1)}{f(y_1)} \tag{17}$$

As ϕ_1 is the known angle:

$$\phi_1 = \tan^{-1}(C_L / C_D), \tag{18}$$

it is seen that Eq. (17) represents a definite functional relation between the (still unknown) quantities, y_1 and ϕ_o. This relation can be written explicitly as:

$$\phi_o = 2\tan^{-1}\left[\left(\tan\frac{\phi_1}{2}\right) e^{-cF(y_1)/f(y_1)}\right] \tag{19}$$

In the following, it is convenient to regard this relation as a known graph relating y_1 to ϕ_o and to represent it symbolically as:

$$\phi_o = \phi_{o,a}(y_1) \tag{20}$$

By virtue of Eq. (20), Eq. (14) may be regarded as a relation between the three quantities, ϕ, y and y_1, and conveniently written as:

$$\tan\frac{\phi}{2} = \left(\tan\frac{\phi_o}{2}\right) g(y, y_1) \tag{21}$$

where:

$$g(y, y_1) = e^{cF(y)/f(y_1)} \tag{22}$$

Now

$$\left.\begin{aligned}\csc\phi &= \frac{1}{2}\left(\tan\frac{\phi}{2} + \cot\frac{\phi}{2}\right)\\ &= \frac{1}{2}\left(g\tan\frac{\phi_o}{2} + g^{-1}\cot\frac{\phi_o}{2}\right)\end{aligned}\right\} \tag{23}$$

Hence,

$$\begin{aligned} s &= \int_0^y \csc\ \phi\ dy \\ &= \frac{1}{2}\left[\left(\tan \frac{\phi_o}{2}\right) G_1 (y, y_1) + \left(\cot \frac{\phi_o}{2}\right) G_{-1} (y, y_1) \right] \end{aligned} \tag{24}$$

where:

$$\left.\begin{aligned} G_1 (y, y_1) &= \int_0^y g\ dy = \int_0^y e^{c F(y)/f(y_1)}\ dy \\ G_{-1} (y, y_1) &= \int_0^y g^{-1}\ dy = \int_0^y e^{-c F(y)/f(y_1)}\ dy \end{aligned}\right\} \tag{25}$$

Finally, the total cable length is given by:

$$S = \frac{1}{2}\left[\left(\tan \frac{\phi_o}{2}\right) \Gamma_1 (y_1) + \left(\cot \frac{\phi_o}{2}\right) \Gamma_{-1}(y_1) \right] \tag{26}$$

where $\Gamma_1 (y_1)$ and $\Gamma_{-1} (y_1)$ are the known functions:

$$\left.\begin{aligned} \Gamma_1 (y_1) &= G_1 (y_1, y_1) = \int_0^{y_1} e^{c F(y)/f(y_1)}\ dy \\ \Gamma_{-1} (y_1) &= G_{-1} (y_1, y_1) = \int_0^{y_1} e^{-c F(y)/f(y_1)}\ dy \end{aligned}\right\} \tag{27}$$

Consequently, for a given value of S, Eq. (26) represents another definite functional, or graphical, relation between y_1 and ϕ_o:

$$\phi_o = \phi_{o,\,b}(y_1) \tag{28}$$

The simultaneous solution of Eq. (20) and (28), which must generally be obtained by numerical and/or graphical methods, furnishes the solution of the present problem, i. e. permits determination of ϕ_o and y_1 for prescribed values of ϕ_1 and S. The cable tension can then be found from Eq. (11).

In many cases, it is also desirable to determine the shape of the cable. This curve is, of course, given in intrinsic form, $s = s(\phi)$, by Eq. (21) and (24). Its Cartesian equation is readily shown to be:

$$x = \frac{1}{2}\left[\left(\cot\frac{\phi_o}{2}\right) G_{-1}(y, y_1) - \left(\tan\frac{\phi_o}{2}\right) G_1(y, y_1)\right] \quad (29)$$

Similarly, the horizontal distance between the hydrofoil and the towing vehicle is:

$$x_1 = \frac{1}{2}\left[\left(\cot\frac{\phi_o}{2}\right) \Gamma_{-1}(y_1) - \left(\tan\frac{\phi_o}{2}\right) \Gamma_1(y_1)\right] \quad (30)$$

2.2 COMPUTATION PROCEDURE

Numerical calculations can be performed using the following procedure, wherein the choice of analytical, numerical and/or graphic methods for each step is left to the discretion of the reader:

A. Using Eq. (10), (13), and (18), calculate

   T', c, and ϕ_1.

B. Combine the towing velocity, V_o, with the known fluid velocity profile, $V_s(y)$, (with proper signs), to obtain:

$$f(y) = \left[1 + \frac{V_s(y)}{V_o}\right]^2$$

   which is functionally equivalent to $f(y_1)$.

C. Calculate the function:

$$F(y) = \int_o^y f(y)\, dy,$$

which is functionally equivalent to $F(y_1)$.

D. Calculate the function:

$$\psi_{-1}(y_1) = e^{-c F(y_1) / f(y_1)}$$

E. Using the value of ϕ_1 from Step A, plot the curve of ϕ_o vs. y_1 corresponding to Eq. (19):

$$\phi_o = 2 \tan^{-1} \left[\left(\tan \frac{\phi_1}{2}\right) \psi_{-1}(y_1) \right]$$

F. Calculate the two families of functions:

$$g(y, y_1) = e^{c F(y) / f(y_1)}$$

and its reciprocal, $g^{-1}(y, y_1)$

G. Calculate the two functions:

$$\Gamma_1(y_1) = \int_o^{y_1} g(y, y_1)\, dy$$

$$\Gamma_{-1}(y_1) = \int_o^{y_1} g^{-1}(y, y_1)\, dy$$

H. Using the known value of S, use Eq. (26) to plot ϕ_o vs. y_1. (Note that for fixed y_1, Eq. (26) is a quadratic equation for $\tan \frac{\phi_o}{2}$).

I. Superpose the two curves of ϕ_o vs. y_1 obtained in Steps E and H. Their point of intersection determines the desired values of y_1 and ϕ_o.

J. Using the latter value of y_1, determine the numerical value of $f(y_1)$ from Eq. (7) and thus the cable tension, T, from Eq. (9).

K. To find x_1, use the final values of y_1 and ϕ_o in Eq. (30).

L. To find the cable curve, use the final value of y_1 to calculate the functions G_1 and G_{-1} defined by Eq. (25) and combine these according to Eq. (29), using the final value of ϕ_o.

2.3 NUMERICAL EXAMPLE

To illustrate the preceding analysis, calculations were made for a hydrofoil-cable system towed through a fluid having a constant velocity gradient, i.e. a linear velocity distribution. Three different gradient values were assumed while all other system characteristics, including towing speed, were kept constant, as follows:

Cable:	C_R = 1.2, D = 1 in., S = 400 ft.
Hydrofoil:	C_L A = 20 ft.^{2}, C_D A = 3-1/3 ft.^{2}
Medium:	Sea Water, ρ = 2 slugs/ft.^{3}
Towing Speed:	V_o = 5 knots
Fluid Velocity:	V_s = by, b = 0, 0.5, 1.0 knots/100 ft.

The cable and hydrofoil characteristics yield the values:

$$c = 0.493 \text{ per 100 ft.}$$

$$\phi_1 = 80° \; 32'$$

and the basic functions are:

$$f(y) = (1 + ay)^2, \quad a = b/V_o = 0,\ 0.1,\ 0.2 \text{ per 100 ft.}$$

$$F(y) = y\left(1 + ay + \frac{a^2}{3} y^2\right)$$

Following the procedure outlined above, and using numerical methods in conjunction with an IBM 610 digital computer to evaluate all of the required integrals, the pair of curves (ϕ_o vs. y_1) for each gradient value was obtained. All of these are shown in figure 2.

The resulting solutions for ϕ_o and y_1, as well as the corresponding values of x_1 and T have been plotted against the gradient parameter, b, in figure 3. It is seen that for the assumed system, the effect of this type of velocity distribution is to elevate the hydrofoil with respect to the towing vehicle, thus increasing the cable tension. The shapes of the cable curves were not calculated.

SECTION 3

DISCUSSION

The following facts may now be noted:

1. UNIFORM STREAM

It is readily shown that the present method correctly yields the known solution for the case of the uniform stream, wherein the cable has the shape of a catenary curve. (Reference 1.)

2. RELATED PROBLEMS

By minor variations of the present method, i. e. by use of different types of cross-plots, the relations established above can be used to solve any problem wherein any two of the four quantities: ϕ_o, ϕ_1, y_1, S, have to be determined from known values of the other two.

3. MORE GENERAL CASES

The present (i. e. cross-plotting) method can be extended readily to solve more general cases such as those involving fluid friction and finite cable weight. Evidently, however, the labor involved in obtaining numerical solutions increases rapidly as the physical system becomes more complex.

SECTION 4

CONCLUSIONS

1. For the two-dimensional hydrofoil-cable system, towed in a straight line at constant speed in a non-uniform stream, the towing problem cannot generally be solved in an explicit manner, even when only normal hydrodynamic forces on the cable and weightless elements are assumed. Instead, the problem has to be solved by an indirect method, i. e., by simultaneous (numerical, graphical, etc.) solution of two transcendental equations.

2. The labor involved in obtaining numerical solutions is appreciable and would be substantially increased in the more general cases involving fluid friction and gravity forces.

3. With regard to the ultimate problem involving steady turning in three dimensions, it is concluded that those aspects relating to the non-uniform flow condition do not offer any inherent mathematical difficulty, but will certainly add considerable labor in obtaining numerical solutions.

REFERENCES

1. Pode, L: "Tables for Computing the Equilibrium Configuration of a Flexible Cable in a Uniform Stream", TMB Report 687, March, 1951.

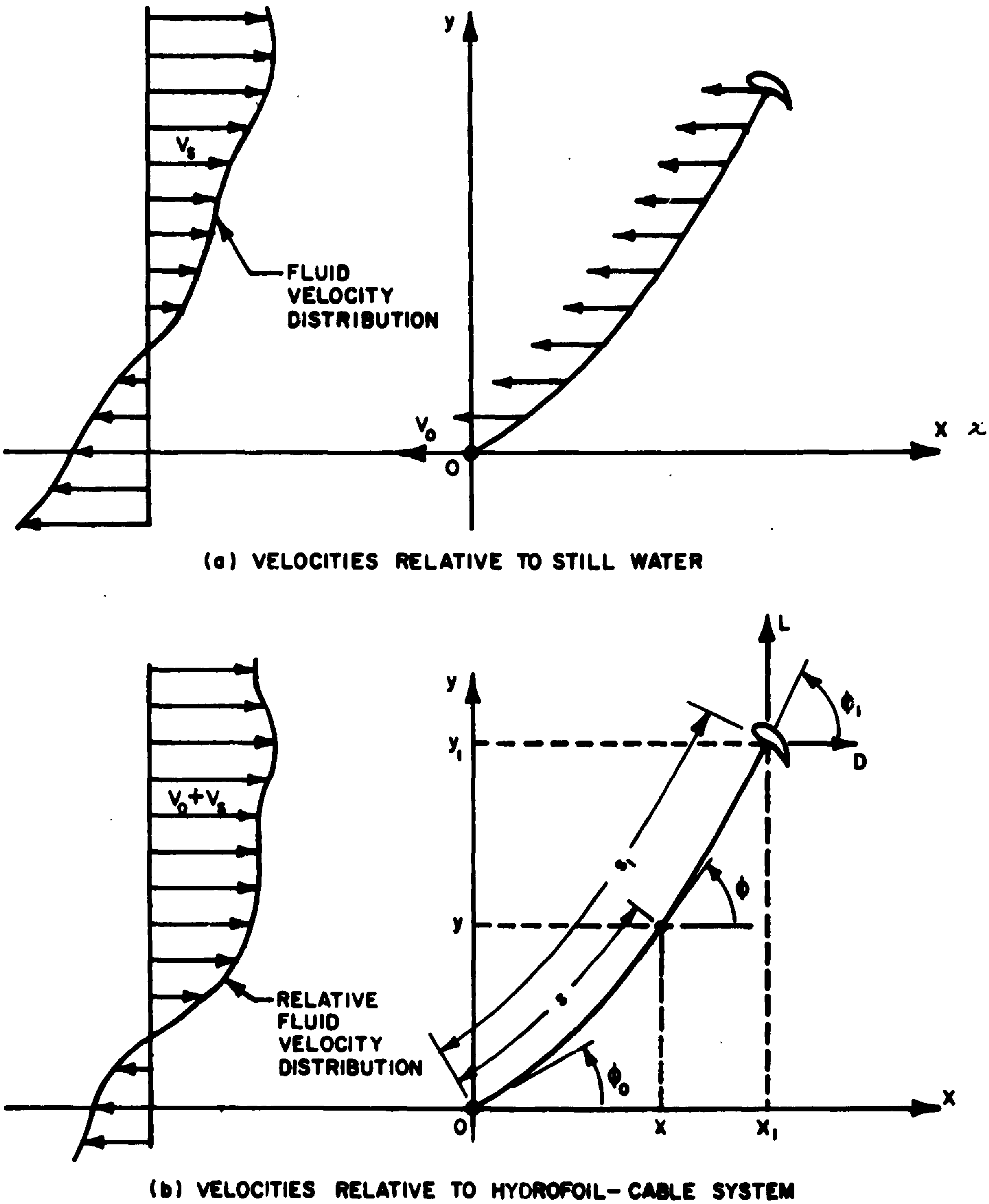

FIGURE 1. SYSTEM CONFIGURATION, ILLUSTRATING ABSOLUTE AND RELATIVE VELOCITY RELATIONS

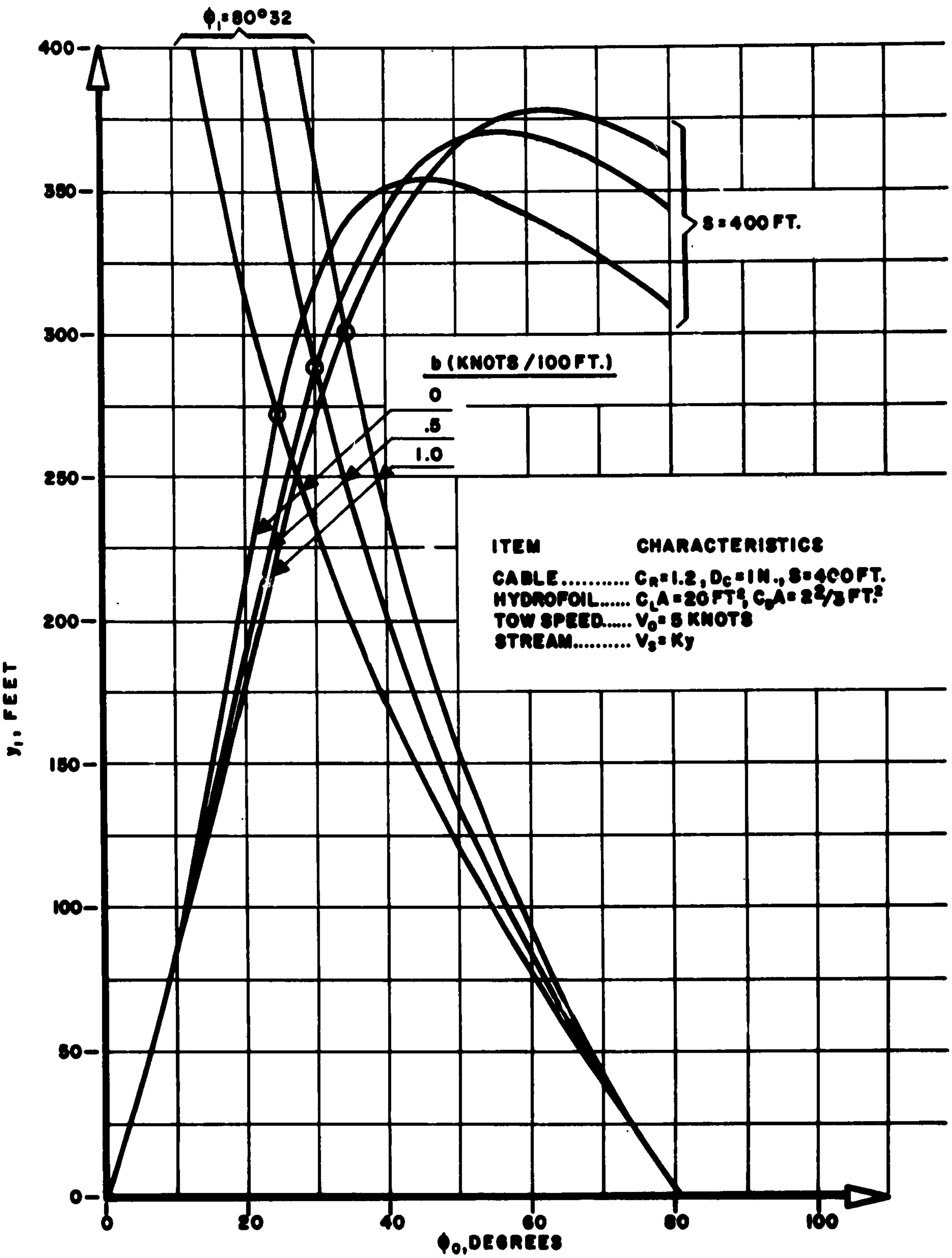

FIGURE 2. TOTAL CABLE HEIGHT VS. CABLE ANGLE AT TOWING VEHICLE

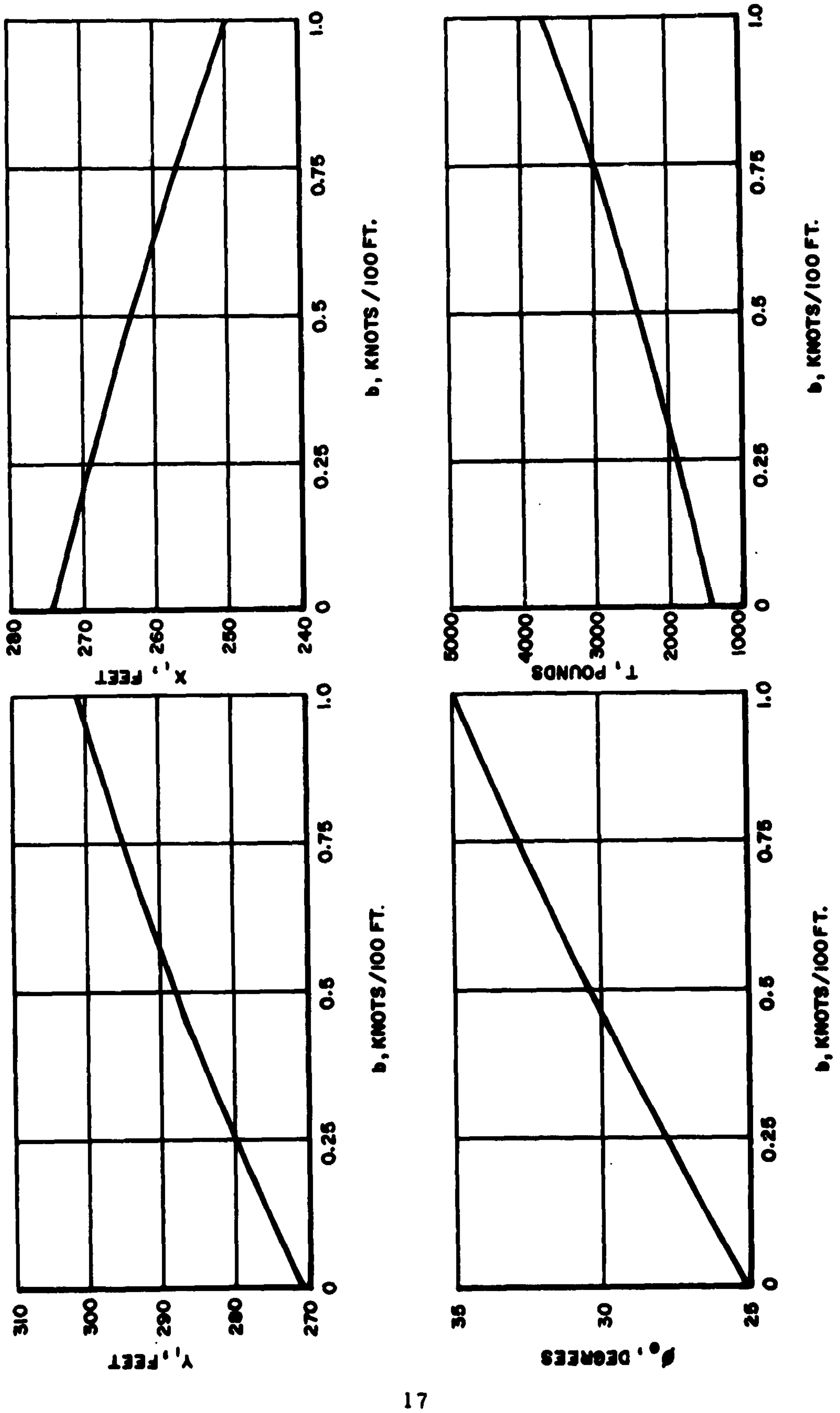

FIGURE 3. SYSTEM GEOMETRY AND TENSION VALUES
VS
GRADIENT MAGNITUDE